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Methods of analysis of precooling data

The ultimate objective of food precooling studies is to provide data that will make possible improved precooling systems. Since scientists who conduct research studies do not directly design cooling systems it is necessary to provide a body of information and methodology for refrigeration engineers who design the cooling systems. The needs of the precooling industry, therefore, are threefold: a procedure for analyzing cooling data; a procedure for using cooling data to design refrigeration systems; and the accumulation of basic data, product physical properties, product thermal properties, and product-refrigerator heat transfer properties so the necessary information is available for cooling system designs to be carried out in an engineering fashion.

The objectives of this paper are to bring together in one place a discussion of the use of the several methods that are available for the evaluation and correlation of cooling data; to compare these methods with the derived equations for transient conduction heat transfer on the basis of precision, difficulty or simplicity of use and flexibility, from both the standpoint of correlating research data and design of cooling systems; to examine these methods in regard to the factors that are important in making a cooling data or cooling rate design analysis; and, most important, to draw conclusions and make recommendations regarding procedures to use in the analysis of cooling rate data.

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The problem we are attempting to resolve is not one of developing a method for analysis of cooling data, but selecting the method that will allow refrigerating engineers and food scientists rapidly and effectively to advance our knowledge in the area of precooling. It is important to point out early in this discussion that rigorous analysis of the basic heat transfer problem has already been completed. The three equations below (from Jakob, 1949) are the derived solutions for transient conduction heat flow into or out of three representative shapes.

Infinite plate or slab: root equation $N_{Bi} = \beta_i \tan \beta_i$

$$\frac{(T - T_1)}{(T_o - T_1)} = \sum_{i=1}^{\infty} \frac{2 \sin \beta_i}{\beta_i + \sin \beta_i \cdot \cos \beta_i} e^{\frac{-\beta_i^2 \alpha t}{a^2}} \cos \beta_i \frac{x}{a}$$

Infinite cylinder: root equation $N_{Bi} = \frac{\beta_i J_1(\beta_i)}{J_o(\beta_i)}$

$$\frac{(T - T_1)}{(T_o - T_1)} = \sum_{i=1}^{\infty} \left(\frac{2}{\beta_i}\right) \frac{J_1(\beta_i)}{J_o^2(\beta_i) + J_1^2(\beta_i)} e^{\frac{-\beta_i^2 \alpha t}{r^2}} J_o\left(\beta_i \frac{x}{r}\right) \tag{2}$$

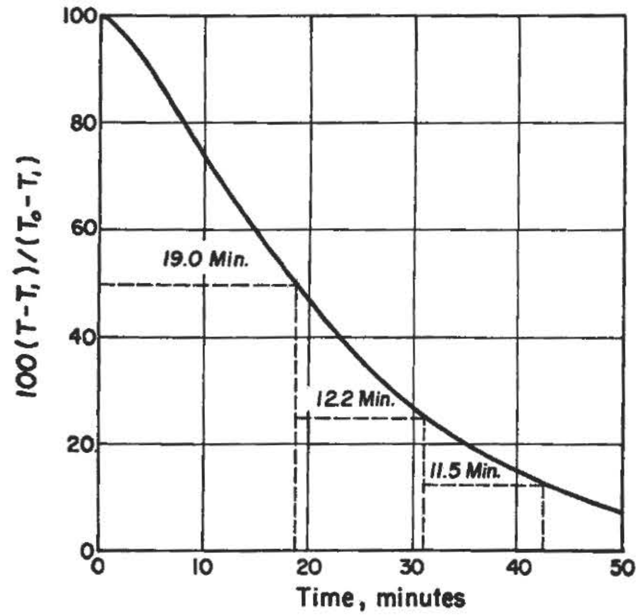

Fig. 1 Method of calculating half-cooling times

Sphere: root equation $N_{Bi} = 1 - \beta_1 \cot \beta_1$

$$\frac{(T - T_1)}{(T_o - T_1)} = \sum_{i=1}^{\infty} 2\,\frac{\sin\beta_1 - \beta_1 \cdot \cos\beta_1}{\beta_1 - \sin\beta_1 \cdot \cos\beta_1}\, e^{\frac{-\beta_1^2 \alpha t}{r^2}}\, \frac{\sin\beta_1 \frac{x}{r}}{\beta_1 \frac{x}{r}} \qquad (3)$$

In this report we are directing our attention to the problem of selecting the method of analysis in the gray area between the rigorous (white) derived solution on one hand and perhaps an arithmetic temperature vs. time graph on the other hand.

If we are to move toward a uniform method of analysis of cooling data, it is prerequisite to outline objectives that the method must satisfy; listed below are some objectives that seem appropriate.

I. Primary objectives for methods of analysis of cooling data:

1. The method should be applicable to a wide range of conditions from slow air cooling to rapid liquid immersion or hydrocooling.
2. The method should be as adaptable to predicting cooling times as to analyzing and correlating cooling time data.
3. The method should be sufficiently convenient and accurate for refrigeration engineers in the field to accept and use.

II. Secondary objectives for methods of analysis of cooling data:

1. The method should have physical significance.
2. The method should have a minimum number of parameters and certainly no more variables or parameters than there are dimensionless numbers in the system.
3. The method should assist food scientists and refrigeration engineers in measuring the thermal and physical properties of food materials.

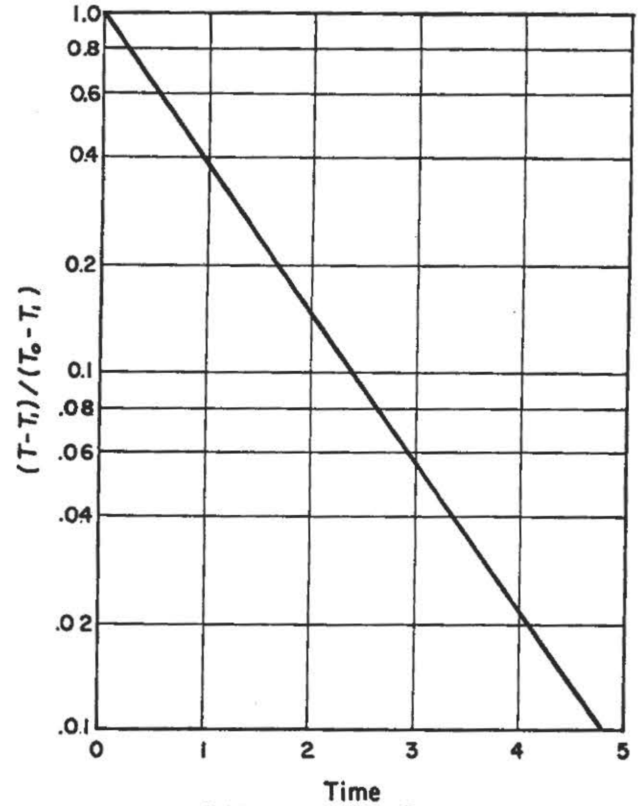

Fig. 2 A typical Newtonian cooling curve

METHODS THAT HAVE BEEN USED TO ANALYZE AND CORRELATE COOLING DATA

In this discussion, we have subdivided the cooling literature into methods using an arithmetic plot and methods using an exponential plot of the temperature-time data.

Methods Based on an Arithmetic Graph of Temperature vs. Time Data

HALF-COOLING TIMES

Two widely used methods of reporting cooling data, the half-cooling time (Richard, 1953; Thevenot, 1955; Hicks, 1955; Guillou, 1960; Sainsbury, 1961, among others) and the cooling rate (Pentzer in Thevenot, 1955; Sainsbury, 1951; Guillou, 1960) also known as the cooling coefficient and the specific speed (Plank as in Thevenot, 1955) use an arithmetic plot of either temperature or per cent initial temperature difference versus time (Fig. 1). These methods are based on the assumption that the product temperature is always uniform throughout.

If the properties and cooling temperature remain constant, the time required to reduce the product temperature to ½ its former value becomes constant. This is the half-cooling time Z. The half-cooling time may be applied readily to determine the time required for a 75% temperature reduction, that is, two successive half-cooling times Z, or any other multiple of Z.

COOLING RATE

When the cooling medium temperature is constant, the cooling coefficient C_r or specific speed σ, which are defined as being equal to the temperature reduction divided by the logarithmic mean temperature difference and

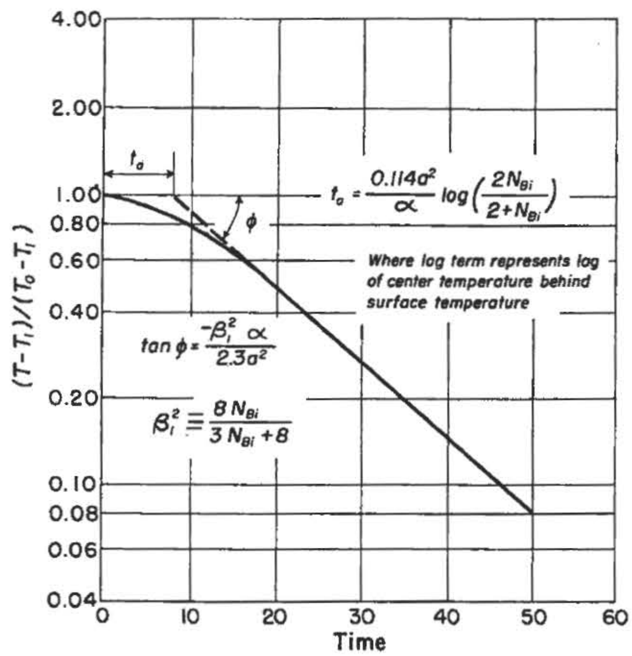

Fig. 3 A cooling curve analyzed according to Backstrom

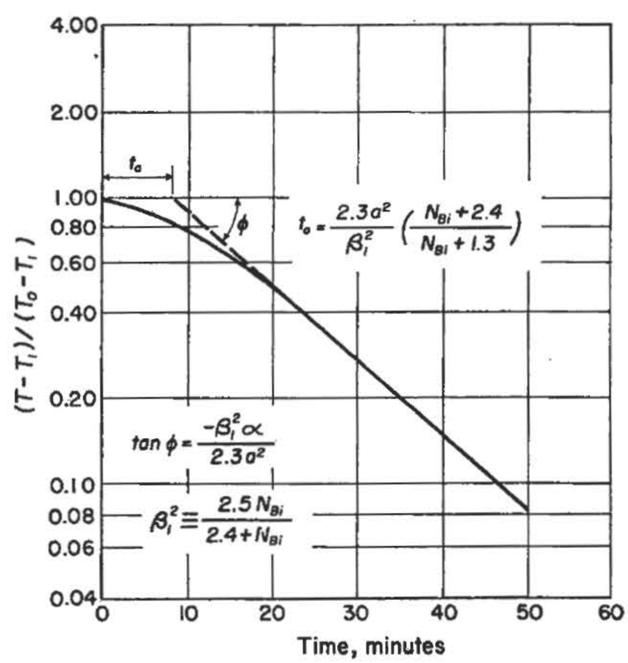

Fig. 4 A cooling curve analyzed according to Rutov

cooling time, may be related to the half-cooling time by the relation:

$$C_r Z = \ln 2 = 0.693 \qquad (4)$$

When the cooling medium temperature is constant, either an algebraic solution or a graphical (semi-logarithmic plot, Sainsbury, 1961) is employed to find the cooling time for a given temperature reduction or the reduction achieved in a given period. When the cooling temperature is not constant, the coolant temperature must be prescribed so that the cooling process can be uniformly divided into more nearly constant temperature periods for calculating the temperature reduction for each period.

Calculation of either the half-cooling time or the cooling rate is straightforward; only the cooling temperature and the product temperature at two known times are required. Both of these methods of analysis use only one parameter, hence design calculations are equally simple.

MASS-AVERAGE TEMPERATURE USING A POLYNOMIAL FITTED TO DATA

Smith and Bennett (1962) developed a procedure whereby, with the aid of an electronic computer, they could fit an equation to cooling data using a multiple regression analysis in the form of third degree polynomials. Using this technique, they were able to develop both mass-average and central temperature curves for various sized fruits. They presented their cooling data in the form: $(T - T_1)/(T_o - T_1)$ vs. time.

Methods Based on a Semi-logarithmic Plot of Temperature-time Data

All three methods, Newtonian cooling, approximate asymptotic solution and true asymptotic solution are based on the concept that conduction cooling data plotted in the form $\log (T - T_1)$ vs. time is a straight line (Fig. 2).

NEWTONIAN COOLING

Newtonian cooling is based on the relationship

$$\frac{d(T - T_1)}{dt} = -K_1 (T - T_1)$$

which can be resolved into

$$(T - T_1) = (T_o - T_1)\, e^{-K_1 t}$$

or

$$\log (T - T_1) = -Kt + \log (T_o - T_1) \qquad (5)$$

A plot of $\log (T - T_1)$ vs. time is linear and the resulting straight line can be described by the slope K. In the Newtonian case, it is assumed that no lag exists in heat transfer from center to surface. The presence of a time lag between a change in temperature at the surface and at the center of a product requires a modification of the basic equation; Leggett and Sutton (1951) used what they call a "bump" factor to modify the Newtonian equation.

Henderson (1957) reported the effect of air velocity on egg cooling rates in the form $\log [(T - T_1)/(T_o - T_1)]$ vs. t. Data were correlated in terms of the slope and the time required for a 90% reduction in temperature.

The Newtonian cooling model is the basis for analysis of cooling by ice bunkers (Richard, 1953 in Thevenot, 1955) or other constant temperature heat sinks (Andersen, 1959; Gac, 1962) or of cooling loads (Refslund, 1955 in Andersen, 1959) among others.

APPROXIMATE ASYMPTOTIC SOLUTIONS

Two types of approximate asymptotic solutions have been proposed. In the first, the cooling process is partitioned into a time required for the cooling process to

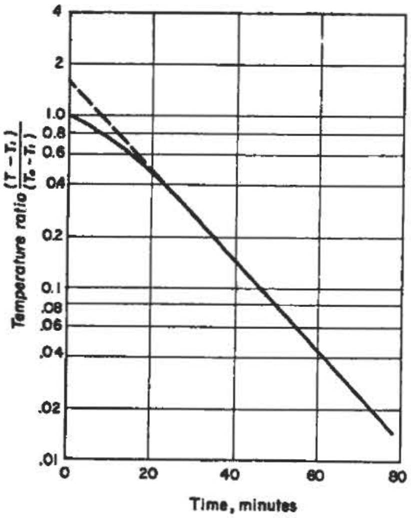

Fig. 5 Cooling curve in convenient form for theoretical use, with temperature scale expressed as a ratio of two temperature differences $T - T_1$ and $T_o - T_1$

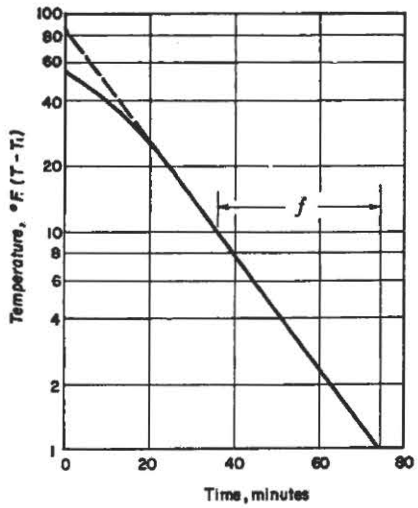

Fig. 6 Cooling curve in convenient form for theoretical use, with temperature scale expressed in degrees difference between processing temperature T_1 and temperature of product

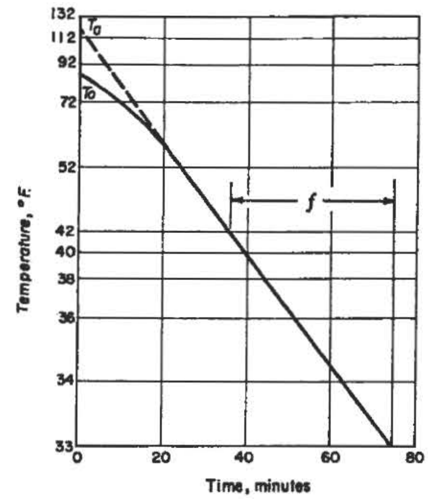

Fig. 7 Cooling curve in convenient form for practical application

affect the center of a product package (Backstrom, 1935, see Fig. 3) or a fictitious time for cooling to become established at its asymptotic rate (Rutov, 1958, see Fig. 4) and a subsequent period in which the cooling process is nearly logarithmetic. In the second (Baehr, 1953) the intercept is defined by the ratio of logarithm (base e) of the constant multiplier of the first term of the series expansion and the square of the first root of the boundary value equation. This ratio $(\ln j)/\beta_1^2$ is defined as equal to 0.10. This is the only approximation of this second method. In the first method, neither graphical nor more exact tabular values of these roots β_1^2 are used.

Noordzij (1962) examined these approximations; he concluded that Rutov's (1958) approach was inconvenient because it was difficult to transform the formulas into simple exponential functions; however, Backstrom's (1935) approximation (for the flat slab), while appreciably simpler than that of Rutov, was less precise.

These three approximate asymptotic solutions, Backstrom, (1935), Baehr, (1953), and Rutov, (1958) all introduce approximations that essentially simplify the estimation of the intercept and slope of the straight line asymptote. Rutov (1958) has shown that neglect of the intercept may introduce errors to predicted cooling times ranging from 1.5 to 30, 1.2 to 20 and 0.8 to 10% for the sphere, cylinder, and slab, respectively, for Biot numbers from 0.1 to 15 when these errors were determined at a 90 per cent reduction in temperature. The methods of Backstrom (1935), Baehr (1953), and Rutov (1958), moreover, are primarily useful for predicting cooling rates rather than correlating cooling data.

TRUE ASYMPTOTIC SOLUTION

Ball (1923) used the equation

$$\log\ (T - T_1) = (-t/f) + \log j\ (T_o - T_1) \qquad (6)$$

to describe the heating and cooling of cans of food in terms of the straight line asymptote to the true heating or cooling curve when conduction heating data are plotted on semi-logarithmic paper. The term f (in min) is the time for the temperature function to decrease by 90%, or from a graphic standpoint the time required for the linear portion of the curve to traverse one log cycle. The j is the ratio of the temperature difference of the asymptote intercept to the true initial temperature difference:

$$j = (T_a - T_1)/(T_o - T_1) \qquad (7)$$

In using the method of Ball (1923), the time-temperature data from conduction heat transfer processes can be plotted as either log $(T - T_1)/(T_o - T_1)$ vs. time, Fig. 5; log $(T - T_1)$ vs. time, Fig. 6; or T, directly on a scale log $(T - T_1) + T_1$ vs. time, Fig. 7, and the best straight line representing the asymptote drawn through the data points. The asymptote can be represented and constructed directly from the two parameters f and j (assuming initial and final temperature data are known). The parameters f and j have significance in terms of the more rigorous solution represented by equation (2); the equations in Table I are applicable to the specific geometric configuration, providing the surface heat transfer resistance is negligible. When the surface heat transfer resistance is large, the method is still applicable but the parameters f and j are now also a function of Biot numbers (Ball and Olson, 1957) as is implied by equations (1), (2) and (3).

Table I. Equations for relating heating or cooling rate f, thermal diffusivity α and dimension of object, also value of j at geometric center. (Data from Olson and Jacksen)

Object	Equation	j at geometric center
1. Brick	$f = 0.933/(1/a^2 + 1/b^2 + 1/c^2)\ \alpha$	2.064
2. Infinite slab	$f = 0.933\ a^2/\alpha$	1.273
3. Finite cylinder	$f = 0.398/(1/a^2 + 0.427/b^2)\ \alpha$	2.040
4. Infinite cylinder	$f = 0.398\ r^2/\alpha$	1.602
5. Sphere	$f = 0.233\ r^2/\alpha$	2.000

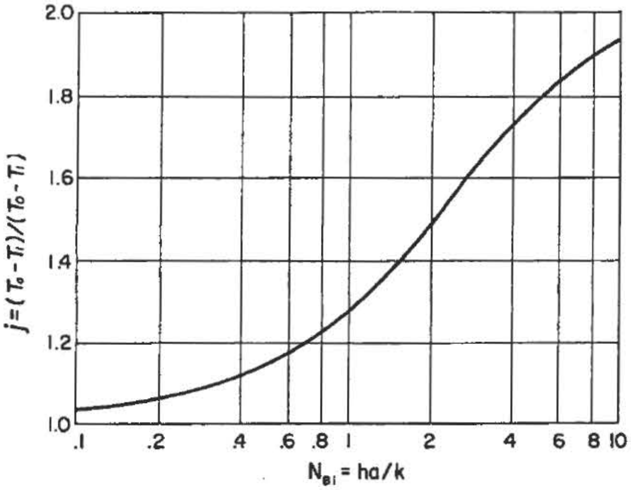

Fig. 8 Effect of Biot number on the lag factor j at the center of an infinite plate

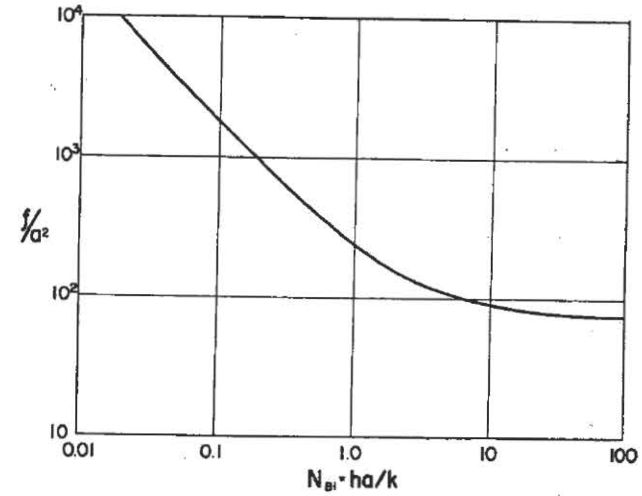

Fig. 9 Effect of Biot number on the 90% cooling time f (min) for an infinite plate with the diffusivity of water at 50 F

METHODS OF ANALYSIS FOR BOTH CORRELATING COOLING DATA AND PREDICTING COOLING RATES

Solution of the equations (1), (2) and (3) is in the form of a series that converges sufficiently fast that at large values of time only the first term is significant. In the case of the infinite plate, the first term approximation is:

$$\frac{(T - T_1)}{(T_0 - T_1)} = \frac{2 \sin \beta_1}{\beta_1 + \sin \beta_1 \cdot \cos \beta_1} e^{\frac{-\beta_1^2 \alpha t}{a^2}} \cos \beta_1 \frac{x}{a} \tag{8}$$

The curve generated by this equation is a straight line when log $(T - T_1)$ or a related form is plotted vs. time. This curve is coincident with the true cooling curve generated from equation (1) at large values of time. It is this fundamental relationship of semi-log temperature plot and a straight line cooling curve that provides the basis for plotting transient conduction heat transfer data in some form of log $(T - T_1)$ vs. time and drawing a straight line through the points.

Equation (8) can be divided into three interesting parts. The term $(2 \sin \beta_1)/(\beta_1 + \sin \beta_1 \cdot \cos \beta_1)$ is the intercept of the straight line asymptote when the temperature at the center of an infinite plate is being considered. The term $\cos \beta_1$ (x/a) is 1.0 at the midpoint where $x = 0$, this term adjusts the intercept for cooling conditions measured at a distance x from the coordinate axis (the center of the infinite slab is at the coordinate axis). The exponential part of the equation contains the time function, also the solution to the root equation (a function of the Biot number), α the thermal diffusivity and a^{-2} (interpreted as the inverse of the half-thickness squared). In a plot of log $(T - T_1)$ vs. time $-\beta_1^2\alpha/2.303a^2$ is the true slope ($\tan \varphi$) of the straight line asymptote.

In Table II, we observe that the solutions of Ball and Olson (1957) are the first term approximations of the true conduction heat transfer equations (1), (2) and (3). Backstrom (1935) and Rutov (1958) recommend similar solutions as far as the slope is concerned; however, only Baehr (1953) uses the true root equation, whereas Backstrom (1935) and Rutov (1958) use approximations in their root equations. The cooling rate, half-cooling time, Newtonian and Leggett and Sutton (1951) and Henderson (1957) methods all use slope values derived from experimental data and not tied in with product properties or dimensions. Ball and Olson (1957) use the first term approximation whereas Baehr (1953), Backstrom (1935) and Rutov (1958) all use approximations in calculating the intercept. The intercept is considered to be 1.0 for the cooling rate, half-cooling time and Newtonian cooling methods.

The Backstrom (1935) method which divides the cooling process into a time $0.114\ a^2/\alpha$ required for the cooling to affect the center of a slab, and a logarithmic period thereafter has been simplified by Andersen (1959). Andersen has plotted the temperature reduction as a function of dimensionless time $(\alpha t/a^2)$ and the Biot number. This approach is straightforward but only a solution for the infinite plate has been worked out to date. Noordzij (1962) has shown that the Backstrom (1935) approximation yields cooling times which are too low for the still or forced air cooling region. The Rutov (1958) approach also partitions the cooling period. The first period is now the period required to establish the asymptotic cooling rate. The principal advantage of the Rutov method is that it is more precise in the low Biot number range than the Backstrom method and no graph or table of roots is required. The Rutov method does not appear (from Noordzij, 1962) sufficiently precise (within 5%) for Biot numbers greater than 15, which corresponds to moderately fast forced air cooling.

The Baehr (1953) method seems to be as precise as the Backstrom (1935) or Rutov (1958) methods but requires a table of roots. If a table of roots is available, it seems preferable to use the method of Ball and Olson (1957).

Viewing the methods summarized in Table II from the cooling rate design point of view, the method of Ball and Olson is the most accurate and most difficult to use with the methods of Baehr (1953), Rutov (1958) and then Backstrom (1935) in order of decreasing precision but also decreasing complexity. Studies are being con-

Table II. Comparison of cooling curve parameters at center of product

Method	Geometry	Root Equation	Intercept*	Slope ($\tan\phi$)
Cooling rate	all	———	1	$\frac{-C_r}{2.303}$
Half-cooling time	all	———	1	$\frac{+\log(1/2)}{Z}$
Newtonian cooling	all	———	1	$\frac{-hA}{2.303 C_p \rho V}$
Leggett and Sutton	all	———	Bump factor	$\frac{-K}{2.303}$
Backstrom	slab	$\beta_1^2 \equiv \left(\frac{+8N_{Bi}}{3N_{Bi}+8}\right)$	$\left(\frac{2N_{Bi}}{2+N_{Bi}}\right) e^{-0.114\beta_1^2}$	$\frac{-\beta_1^2 \alpha}{2.303a^2}$
Rutov	slab	$\beta_1^2 \equiv \left(\frac{2.5N_{Bi}}{2.4+N_{Bi}}\right)$	antilog $\left(\frac{0.11N_{Bi}}{1.3+N_{Bi}}\right)$	$\frac{-\beta_1^2 \alpha}{2.303a^2}$
Rutov	cylinder	$\beta_1^2 \equiv \left(\frac{6N_{Bi}}{2.85+N_{Bi}}\right)$	antilog $\left(\frac{0.22N_{Bi}}{1.7+N_{Bi}}\right)$	$\frac{-\beta_1^2 \alpha}{2.303r^2}$
Rutov	sphere	$\beta_1^2 \equiv \left(\frac{10.3N_{Bi}}{3.2+N_{Bi}}\right)$	antilog $\left(\frac{0.33N_{Bi}}{2.1+N_{Bi}}\right)$	$\frac{-\beta_1^2 \alpha}{2.303r^2}$
Baehr	all	Same as those of Ball and Olson below.	$e^{0.10\beta_1^2}$	$\frac{-\beta_1^2 \alpha}{2.303(b^2 \text{ or } r^2)}$
Ball	all	———	j	$\frac{-1}{f}$
Ball and Olson	slab	$N_{Bi} = \beta_1 \tan \beta_1$	$\frac{2 \sin \beta_1}{(\beta_1 + \sin \beta_1 \cos \beta_1)}$	$\frac{-\beta_1^2 \alpha}{2.303b^2}$
Ball and Olson	cylinder	$N_{Bi} \cdot J_o(\beta_1) = \beta_1 J_1(\beta_1)$	$\left(\frac{2}{\beta_1}\right)\left(\frac{J_1(\beta_1)}{J_o^2(\beta_1) + J_1^2(\beta_1)}\right)$	$\frac{-\beta_1^2 \alpha}{2.303r^2}$
Ball and Olson	sphere	$N_{Bi} = 1 - \beta_1 \cot \beta_1$	$2\left(\frac{\sin \beta_1 - B_1 \cos \beta_1}{B_1 - \sin \beta_1 \cos \beta_1}\right)$	$\frac{-\beta_1^2 \alpha}{2.303r^2}$

* Intercept value $= \frac{T_a - T_1}{T_o - T_1}$

ducted at the present time to simplify the use of the method of Ball and Olson by graphically relating the Biot number and the intercept (Fig. 8), and the Biot number and the ratio f/a^2, Fig. 9.

FACTORS TO BE CONSIDERED IN SELECTING A METHOD

For this discussion, we have divided the problem in three parts: (1) method of plotting data, (2) describing the direction of the curve, and (3) treatment of the initial lag phase. Listed below are some of the alternatives that are available in these three categories:

1. Method of plotting data including both coordinate system and scale.
   a. Arithmetically temperature vs. time
   b. Arithmetically $(T - T_1)/(T_o - T_1) \times 100$ vs. time
   c. log $(T - T_1)/(T_o - T_1)$ vs. time
   d. log $(T - T_1)$ vs. time
   e. T on a scale $(\log (T - T_1) + T_1)$ vs. time
2. Describing the direction of the curve.
   a. True slope ($\tan \varphi$)

Table III. Factors to convert from one cooling rate to another

$$f = 2.303/C_r$$
$$C_r = 0.693/Z$$
$$f = 3.32Z$$

b. f suggested by Ball (1923)
c. Half-cooling time
d. Cooling coefficient
e. K (Leggett and Sutton, 1951)
f. Time constant
3. Treatment of the initial lag phase.
a. Ignored (considered equal to 1.0)
b. Assumed to be a variable depending on shape, position and N_{Bi}
c. Incorporated in the first half or 90 per cent cooling time

Method of Plotting Data

The selection of the coordinate system as well as the scale system is important from the standpoint of the utility of analysis method. In discussing the choice of a coordinate system, Ball and Olson (1957) wrote, "If the time-temperature data are plotted on rectangular coordinate paper, the resulting curve may be studied for obvious irregularities, maxima or minima, and comparison with other data similarly plotted. However, if the curves are plotted on semi-logarithmic paper, many characteristics which are undetectable on cross-section paper become obvious on inspection of the graph. It will be found that this method of plotting the curves greatly increases the amount of information which may be derived from the data." Simplicity is probably the chief attribute of the arithmetic temperature-time plot; this is an important consideration because of the need for simplicity from the user's point of view.

The scale system of the semi-log plot can either simplify or confuse; in Figs. 5, 6 and 7, three semi-logarithmic scale arrangements are illustrated using the same data. Using the f of Ball (1923) as the measure of direction and the j of Ball (1923) as the intercept, the equation of the asymptote in Fig. 5 is:

$$\log \frac{(T - T_1)}{j\,(T_o - T_1)} = \frac{-t}{f}$$

in Fig. 6,

$$\log (T - T_1) = \frac{-t}{f} + \log j\,(T_o - T_1)$$

and in Fig. 7,

$$\log (T - T_1) + T_1 = \frac{-t}{f} + \log j\,(T_o - T_1) + T_1$$

We believe that engineers and scientists working with practical problems will have considerably more feel for the data and those not regularly working in the area will be able to use the method with less study when the data are plotted as in Fig. 7. Therefore, we shall describe in detail the method used to plot the cooling data as in Fig. 7. Since the method used for Figs. 5 and 6 is also important, we will discuss this also, but briefly. The initial ingredient in plotting the exponential cooling curve is a quantity of 2 or 3-cycle semi-logarithmic paper. (We use tracing graph paper to permit direct comparison of cooling curves, however, drawing graph paper can be used.) The use of semi-logarithmic paper means that in the case of Fig. 5 we are plotting $(T - T_1)/T_o - T_1)$ vs. time; in Fig. 6 we are plotting $(T - T_1)$ vs. time; and in Fig. 7, by adding T_1 to our semi-logarithmic scale, we are plotting T vs. time.

The temperature scale in Fig. 7 is constructed by adding the cooling medium temperature to the log scale values. Normally, using 2-cycle paper, the bottom line will have a value of 1.0, the line at the top of the first cycle a value of 10, and the line at the top of the second cycle, 100. If the cooling medium temperature T_1 is 32 F (as is the case in Fig. 7), the bottom line has a value of 33 F, the second line 42 F and the top line 132 F. Cooling time-temperature data can now be plotted directly using the new scale. A straight line is drawn through the data points and the f is the time required for the line to traverse one log cycle. The temperature value where the straight line intersects the y-axis is T_a and the intercept ratio $j = (T_a - T_1)/(T_o - T_1)$.

Describing the Direction of the Curve

In discussing this aspect of cooling data analysis, we shall direct our attention to the exponential graphs illustrated in Figs. 5, 6 and 7. At this point we are all in agreement that the asymptote to the conduction cooling curve is a straight line, and when time values are large the asymptote and the cooling curve are coincident. Therefore, the several methods of describing the direction of this line: f of Ball (1923), half-cooling time Z, the cooling coefficient C, the time constant τ, or the K of Leggett and Sutton (1951), all are measures of the true slope ($\tan \varphi$). The various equalities of these several terms are listed in Table III.

The selection of a measure of the direction of the straight line asymptote should be made on the basis of convenience, since we anticipate that this value will eventually be the tool used to construct cooling curves to predict cooling times. The f-value of Ball (1923), the time in minutes for the curve to decrease by 90 percent or to traverse one log cycle is convenient; as is the half-cooling time Z, the time for the curve to decrease by 50%; other terms also can be used. We prefer the 90% reduction time, f, of Ball (1923) because we have used it for a long period of time and this terminology and procedure is used throughout the processed food industry.

We believe that the most important aspect of cooling curve analysis lies in relating the direction of the asymptote to the size of the body being cooled and to the external heat transfer characteristics represented by the Biot number. The curve in Fig. 9 relates the ratio f/a^2 and Biot number and this graph makes possible the development of a cooling curve if the external heat transfer characteristic product dimensions and product thermal properties are known.

The semi-logarithmic cooling curve has useful properties; for example, the slope of the asymptotic cooling curve is the same regardless of the point of measurement in the conduction cooling object. If tests are conducted under rigidly controlled conditions, the asymptotic curve can be used to evaluate the thermal diffusivity or the surface heat transfer coefficient. Other graphical systems (see Schneider, 1955 and 1963, McAdams, 1954, Jakob, 1949 among others) have been developed for solution of these and other problems, but few utilize the fact that cooling curves as plotted here can be represented in terms of the slope and lag of a straight line at moderate to long times. Needless to say, the system we are discussing can be used equally well for heating as for cooling.

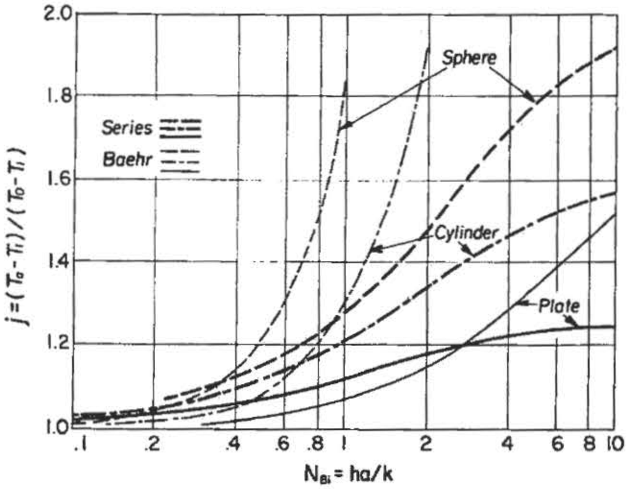

Fig. 10 Lag factor j for various shapes (and its Baehr approximation) as affected by Biot number

Treatment of the Initial Lag Phase

In this phase of the discussion of cooling data analysis, we shall again direct our attention to the exponential graphs in Figs. 5, 6 and 7. In the discussion above it was concluded that the temperature-time points when time is large are coincident with the straight line asymptote. If there are sufficient points to establish a line then the extension of this line will intercept the y-axis. This line will have an intercept value $(T_a - T_1)/(T_o - T_1)$ of 1.0 only if there is no heat transfer lag at the point of measurement. Since we recognize the existence of a cooling lag if the object has significant mass, the existence of variation in intercept must be acknowledged and the value of this intercept reported along with the direction of the asymptote. The error introduced when an intercept value of one is assumed is shown in Fig. 10, which also indicates how the Biot number affects the intercept value.

Ball (1923) suggested the term j to represent $(T_a - T_1)/(T_o - T_1)$ and it is suggested that this or a similar term be used to indicate the cooling lag. The effect of Biot number on the j at the center of a slab is shown in Fig. 8; it can be noted from this curve that when surface resistance is very large, the j at the center approaches one; however, j increases as the surface resistance decreases.

CONCLUSIONS

1. To obtain maximum information, cooling data should be plotted in the form log $(T - T_1)$ vs. time. It is recommended that semi-logarithmic paper be used and that the log scale be $(T - T_1) + T_1$ as in Fig. 7 so temperature can be plotted directly. The linear portion of the curve will include the valuable data when product approaches cooling medium temperature.

2. The intercept of the asymptotic cooling curve is a variable and therefore should be measured and reported as part of cooling data analysis.

3. The value of the intercept factor is a function of the location of the measurement and the surface heat transfer coefficient.

4. Conduction cooling curves can be fully described by a slope and an intercept value.

5. A derived relationship exists between the slope of the linear asymptote and the basic heat transfer conditions; this relationship should be used to obtain maximum information from cooling tests.

6. Changes in product size, thermal properties or surface heat transfer coefficient will change the slope of the linear asymptote.

7. The slope of the linear asymptote for a given object is the same regardless of point of temperature measurement in the object.

8. We prefer the f and j of Ball (1923) as a measure of direction and intercept because we are familiar with these terms and they are used throughout the processed food industry.

NOTATION

A	Surface area of product
2a	Thickness of infinite slab
2b	Width of brick, length of finite cylinder
2c	Thickness of brick
C_p	Specific heat (at constant pressure) of product
C_r	Cooling rate or cooling coefficient
D	Diameter of spherical or cylindrical product
e	Napierian base (= 2.71828 . . .)
f	Time required (in minutes) for the asymptote of the cooling curve to cross one log-cycle, that the time required for a 90% reduction of temperature on the linear portion of the cooling curve (Ball, 1923).
g_c	Gravitational constant
h	Surface heat transfer coefficient
$J_n(\beta_n)$	Nth order Bessel function of first kind for the argument β_n.
j	Lag factor $(T_e - T_1)/(T_o - T_1)$
K	Coefficient used by Leggett and Sutton to denote the rate of cooling
k	Thermal conductivity
L	Characteristic length of product in the direction of fluid flow
N_{Bi}	Biot number $h/[k_p/a]$
N_{Gr}	Grashof number $L^3\rho^2 g_c(T_1 - T_{film})/\mu^2$
N_{Nu}	Nusselt number $h(L \text{ or } D)/k_f$
N_{Pr}	Prandtl number $C_p\mu/k_f$
N_{Re}	Reynolds number $(L \text{ or } D)V\rho/\mu$
r	Radius of sphere or infinite cylinder
T	Temperature; to initial temperature of product T, temperature of coolant or heating media; T_a the apparent initial temperature as defined by the linear portion of the cooling curve, that is, the ordinate value of origin of asymptote of cooling curve. T_f means temperature of fluid that is $(T_s + T_1)/2$ where T_s is the surface temperature
t	Time
x	Distance from center of product to point of measurement
V	Volume of product in Newtonian cooling equation, velocity of fluid in N_{Re}
Z	Half-cooling time
α	Thermal diffusivity that is $k/(C_p\rho)$
β	Volumetric coefficient of expansion of fluid
β_n	Nth root of the boundary equation, or the prescribed approximate boundary equation for a particular shape. The β_n for a slab cylinder or sphere are different
μ	Viscosity of fluid

π Pi ($= 3.1416 \ldots$)

ρ Density

σ Specific speed, as defined by Plank

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