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CORRELATING AND PREDICTING TRANSIENT HEAT TRANSFER RATES IN FOOD PRODUCTS*

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SUMMARY : A new approach to the characterization and presentation of the conduction transient heat transfer system is introduced. A new term, the temperature response parameter, *f*, that has the unique property of incorporating body and system transient heat transfer properties into a single term is described and its use demonstrated. Generally, when the Fourier number $t\alpha/R^2$ is greater than 0.3 the aperiodic cooling curve can be described by the first term approximation and the proposed method of presentation has the advantages of suggesting the power of proportionality of the parameters with respect to the heat flow.

Corrélation et prévision des taux de transmission de chaleur, en régime variable, dans les produits alimentaires.

RÉSUMÉ : *On présente une nouvelle étude sur la définition et la représentation du système de transmission de chaleur en régime variable. On décrit un nouveau terme, le paramètre de réponse à la température,* f, *qui a la particularité de représenter à lui seul les propriétés de transmission de chaleur, en régime variable, d'un corps et d'un système et on précise son utilisation. En général, lorsque le module de Fourier* $t\alpha/R^2$ *est supérieur à* 0,3, *la courbe de refroidissement apériodique peut être représentée par le premier terme de la série et la méthode de représentation proposée à l'avantage de suggérer l'exposant de proportionnalité des paramètres par rapport au flux de chaleur.*

INTRODUCTION

There are five groups of variables that must be considered in the transient heat conduction system : the dimension parameter of the body, R, and the position variable, *r*; the body properties—thermal conductivity, *k*, and thermal capacitance, ρc_p; the external film coefficient, *h*; the time, *t*; and the temperature change of the body, $(T_1-T)/(T_1-T_0)$.

A transient heat conduction system usually consists of two basic thermal resistances, the external resistance, $1/h$, and the internal resistance, R/k. The ratio of the internal to the external resistance, hR/k is the Biot number, N_{Bi}. The N_{Bi} is a positive number with limits between zero (no internal resistance-Newtonian system) and infinity (no external resistance). The solution for the extreme conditions is simpler than the solution for those cases where both resistances enter into the calculations. We can afford to ignore the internal resistance (i.e. $N_{Bi}=0$) when N_{Bi} is less than 0.1 or to ignore the external resistance (i.e. $N_{Bi}=\infty$) when N_{Bi} is greater than 100 even though in practice these extreme N_{Bi} values of zero or infinity are never actually reached.

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Many industrial and practical transient heat conduction problems deal with the evaluation of temperature profiles. The information obtained from the temperature profile can be used for predicting the process time, computing the local or overall heating loads for instantaneous or integrated time, or evaluating the thermal properties. This information may be used in many other cases where prediction of a temperature dependent phenomenon is needed such as in solidification and melting, microbial death and chemical reaction. These aperiodic transient cooling and heating problems are typical of the canning, metallurgical and glass industries. The most common case will be that of a body initially at a uniform temperature, T_0, suddenly exposed to a new constant temperature, T_1. The boundary conditions are :

$$T = T_0 \quad \text{at} \quad t = 0 \quad \text{for all } r$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = 0$$

$$-k\frac{\partial T}{\partial r} = h(T - T_1) \quad \text{at} \quad r = R \quad \text{for } t \geqslant 0$$

The exact solution for the three major one dimensional heat flow geometries, infinite slab, sphere and infinite cylinder having the above boundary conditions are listed in equations A-1, A-17 and A-9. There are a number of graphical presentations such as charts of Gurney-Lurie, Hottel and Williamson-Adams (McAdams, 1954) of equations A-1, A-9, and A-17 for practical use. Most of these charts are plotted as $(T_1 - T)/(T_1 - T_0)$ vs. the Fourier number, $\alpha t/R^2$ with N_{Bi} as the parameter. In the following sections we shall show a new approach for the presentation of the above mentioned equations which in certain cases has advantages over the conventional methods.

Presentation of the method

The exact solution for the infinite plate, the sphere and the infinite cylinder (equations A-1, A-17 and A-9) all have the following form

$$\frac{T - T_1}{T_0 - T_1} = \sum_{i=1}^{\infty} je^{-\beta_i^2 \alpha t/R^2} \tag{1}$$

After enough time has elapsed the series type solution (equation 1), because of its exponential nature, converges rapidly, and all the terms except the first become negligible (equation 2)

$$\frac{T - T_1}{T_0 - T_1} = je^{-\beta_1^2 \alpha t/R^2} \tag{2}$$

Taking the logarithm of both sides of equation 2 we obtain equation 3,

$$\log(T-T_1) = -\frac{t}{f} + \log j(T_0-T_1) \tag{3}$$

which produces a straight line if we plot log $(T-T_1)$ vs t. The parameters of this line are :

$$\text{slope} = \frac{-\beta_1^2\alpha}{2.303\ R^2} = \frac{1}{f}$$

$$\text{intercept} = \log\left[j(T_0-T_1)\right]$$

$$j = \frac{T_a - T_1}{T_0 - T_1}$$

Equation 3 describes the asymptote or the straight line portion of the cooling curve, generated when the logarithm of the difference in temperature of an object, $(T-T_1)$, is plotted vs. time (fig. 1).

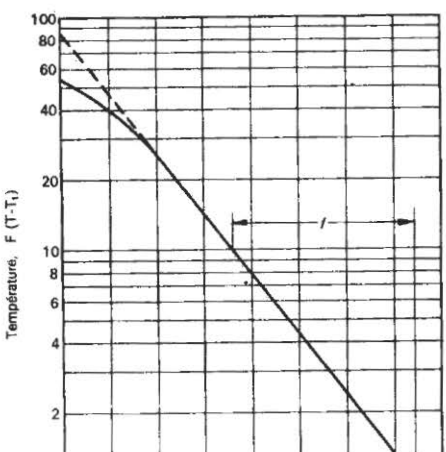

Fig. 1 — Cooling curve in convenient form for theoretical use, with temperature scale expressed in degrees difference between processing temperature T_1 and temperature of product.

Figure 1 can be shown to be identical to the common transient heat transfer charts mentioned above, where $\alpha t/R^2$ is plotted vs. $(T-T_1)/(T_0-T_1)$, when the abcissa is multiplied by α/R^2 and the ordinate divided by $(T_0 - T_1)$ both of which are constants for the system.

Equation 2 which is the first term approximation of the series solution and beyond a certain time is the actual cooling curve can be described by two parameters, the temperature response parameter, f, the direction function, and the j parameter which is the intercept function. The f of the straight line semi-logarithmic cooling curve, is independent of the point of measurement since the slope term does not contain a position variable. The j term, however, does depend on location since it contains the position variable r/R. There are three important j's, center, surface and mass average.

geometric center $\quad j_c = j|_{r=0}$

surface $\quad j_s = j|_{r=R}$

mass average $\quad j_m = \dfrac{1}{m}\displaystyle\int_0^m j\,dm$, where m = mass

The importance of the various j's is that once the straight line semi-logarithmic heating curve is established we can determine the lowest (heating) or highest (cooling) temperature of the body, the surface temperature and the mass average temperature as a function of time by using j_c, j_s, and j_m, respectively.

The temperature response parameter is a very convenient practical term for describing the transient system since by single term having a time dimension we can describe a 90% change in temperature difference on the linear portion of the curve. This term incorporates the thermal properties of the body, its geometric characteristic and the thermal properties of the external system, and describes the transient heat conduction system just as the overall heat transfer coefficient, U, describes the steady-state system. By dimensional analysis of the temperature response parameter (Kopelman, 1966) it was shown that the system can be described by two dimensional groups $f\alpha/R^2$ and N_{Bi}. Solutions for other isotropic regular shapes may be obtained from those of the infinite slab and/or the infinite cylinder for prescribed surface temperature or film coefficients and the same initial and boundary conditions. Composite solutions are the product of the respective individual direction solution (Carslaw and Jaeger, 1959) given by equations A-1, A-9, and A-17, hence for a solid with three dimensional heat transfer.

$$\frac{1}{f_{composite}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \tag{4}$$

$$j_{composite} = j_1 j_2 j_3 \tag{5}$$

The curves in figures 2, 3, 4 and 5 are our method of presenting the first term approximation solution and were shown with tables of values first by

Pflug *et al.* (1965). These figures relate the f and j with the system conditions for the three major one dimensional heat flow geometries, the infinite slab, sphere and the infinite cylinder. (As it was mentioned it is possible to combine these data to make other solutions for other geometries.) Figure 2 relates the ratio $f\alpha/R^2$ for an infinite slab, sphere and infinite cylinder with the Biot number, hR/k. Figures 3, 4 and 5 relate the lag factor j and the Biot number

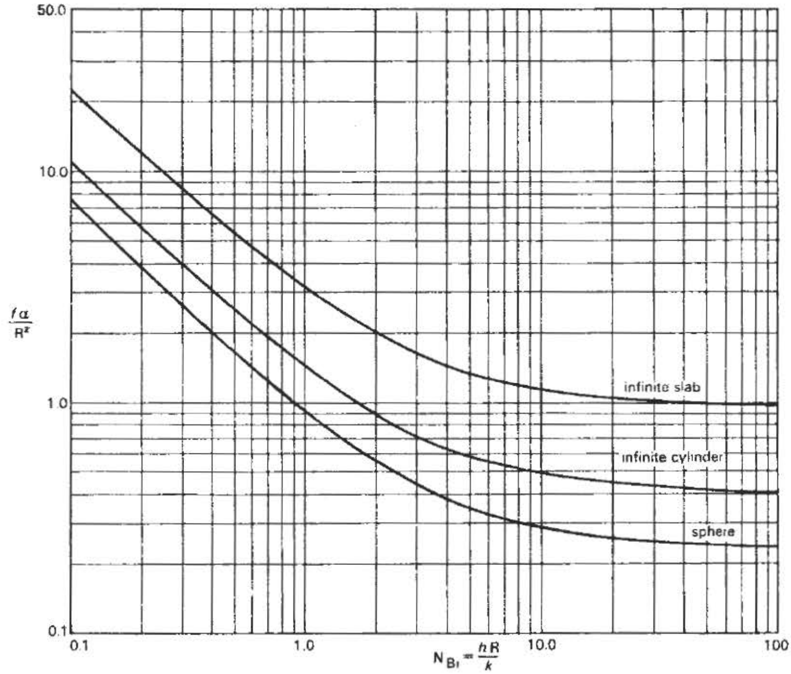

Fig. 2 — N_{Bi} vs $f\alpha/R^2$ for infinite slab, infinite cylinder and sphere.

for the infinite cylinder, sphere, and infinite slab; figure 3 for the center of the object (j_c); figure 4 for the point representing the mean temperature of the object (j_m); and figure 5 for the surface temperature (j_s).

In the evaluation of the transient heat transfer system, where the system properties are constant, we have two variables, time and temperature. These two variables, time and temperature, appear in the conventional method of presentation mentioned above. In our method, by definition, we incorporated these two variables into a system property—the temperature response parameter, f. Through the incorporation of these two variables into a system property, we are able to describe the system as a function only of its properties.

In the conventional method of presentation where the unaccomplished temperature change, $(T-T_1)/(T_o-T_1)$ is plotted vs. the Fourier number $\alpha t/R^2$, a series of lines are generated for each parameter, for example N_{Bi} and position, r/R. For values of $\alpha t/R^2 > 0.3$ these lines are straight and practically indicate a plot of first term approximation. When $\alpha t/R^2 < 0.3$ the lines curve showing that first term approximation does not hold. Regardless of the fact that

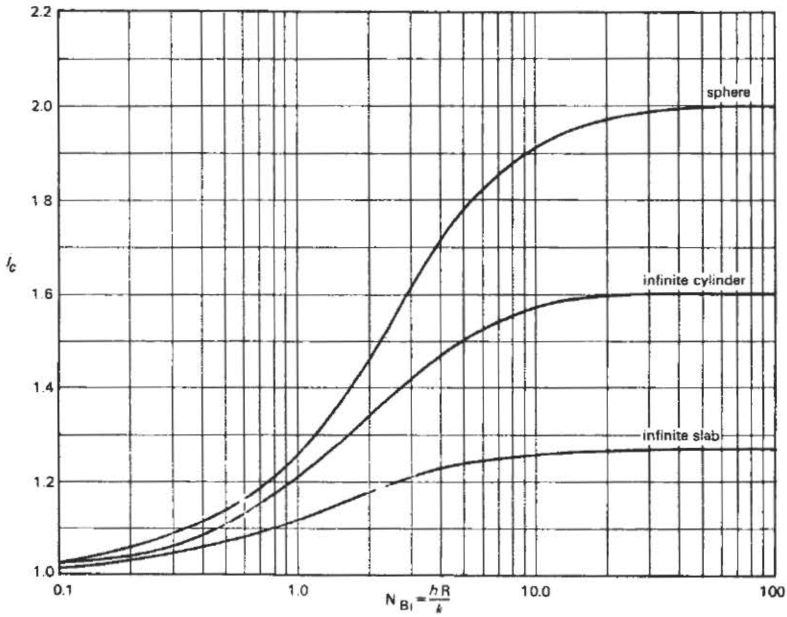

Fig. 3 — j_c vs N_{Bi} for infinite slab, infinite cylinder and sphere.

in this low Fourier number region we cannot accurately read $(T-T_1)/(T_0-T_1)$, it is a marked advantage of these charts to be able to determine, by inspection, whether or not we can use the first term approximation. In our method of presentation, the figures are already based on the assumption that we are dealing with the first term approximation, therefore we must make sure that in our system sufficient time has elapsed so that this assumption is valid. On the other hand, if the Fourier modulus exceeds 0.3, we believe that our method of presentation has advantages, since we can determine which part of the system is controlling by visual inspection of the charts. For example, by inspection of figure 2 we can see that when the $N_{Bi} > 100$ the lines of $f\alpha/R^2$ vs. N_{Bi} approach an asymptotic value which suggests that f is inversely proportional to the thermal diffusivity, directly proportional to the square of the characteristic dimension and independent of the N_{Bi} or the film coefficient h. Thus an attempt to improve the performance of a high N_{Bi} system by improving the film coefficient will not be very successful. It is possible to significantly alter the response of the body by changing the body characteristic dimension. On the other hand in the low N_{Bi} system (fig. 2) the curve becomes a straight line of slope = −1, where the temperature response parameter is directly proportional to the heat capacitance ρC_p and the characteristic dimension, and inversely proportional

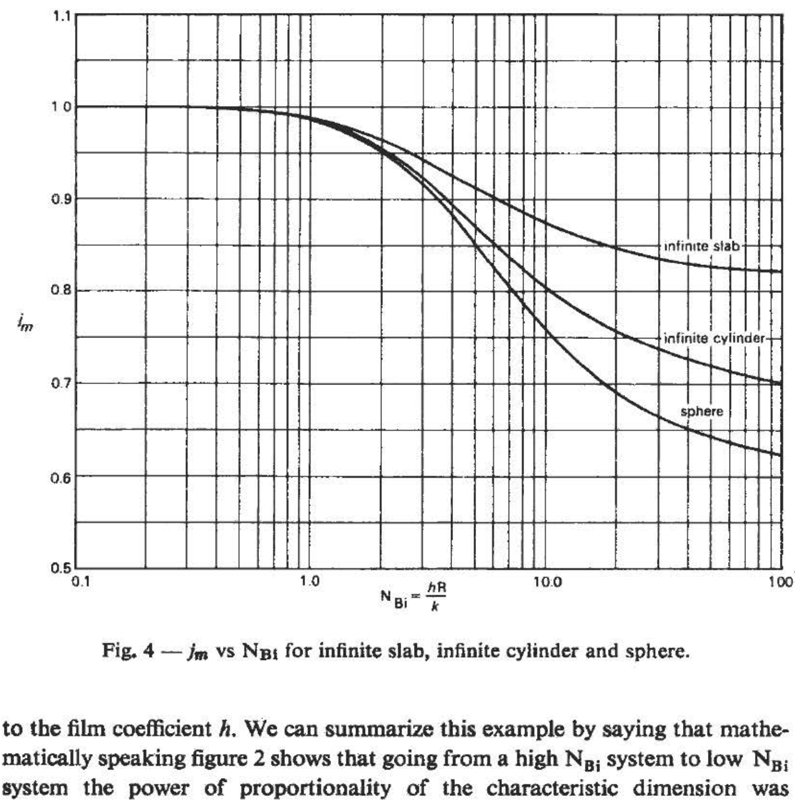

Fig. 4 — j_m vs N_{Bi} for infinite slab, infinite cylinder and sphere.

to the film coefficient h. We can summarize this example by saying that mathematically speaking figure 2 shows that going from a high N_{Bi} system to low N_{Bi} system the power of proportionality of the characteristic dimension was decreased from 2 to 1 while the power of proportionality of the film coefficient was increased from 0 to 1. Similar analysis can be made with respect to the relationship of j with respect to N_{Bi} shown in figures 3, 4 and 5. Kopelman (1966) did a complete analysis of the curves appearing in this presentation. The ability to isolate the power of proportionality of the parameters with respect to heat flow in the various Biot number regions is both important and convenient in the design of an experiment or in the evaluation of the experimental data.

EXAMPLE

A food product in slab form 2 in. × 12 in. × 12 in. (5.08 cm × 30.5 cm × 30.5 cm), $k = 0.25$ Btu/hr ft °F (0.372 kcal/hr m °C), specific gravity = 1.00, $C_p = 0.8$ Btu/lb °F (0.8 kcal/kg °C), is to be cooled from an initial temper-

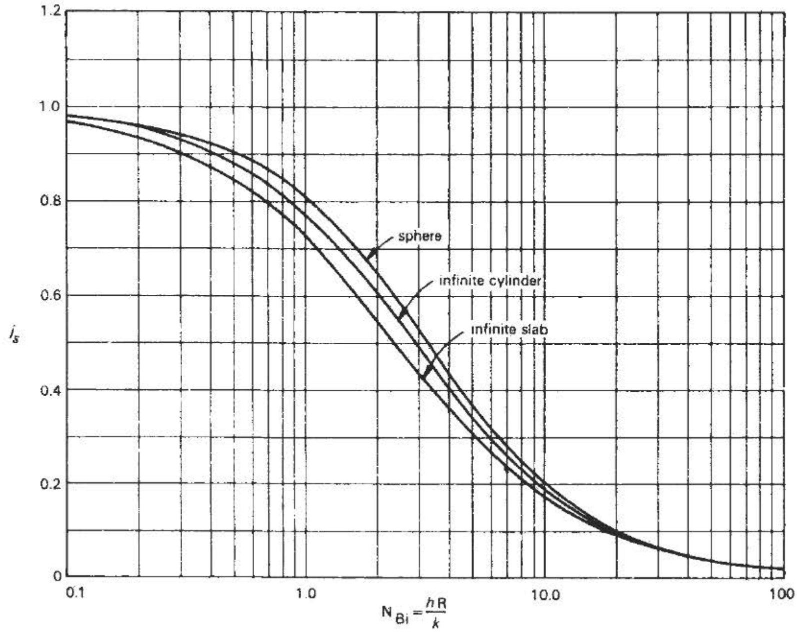

Fig. 5 — j_s vs N_{Bi} for infinite slab, infinite cylinder and sphere.

ature of 100 °F (37.8 °C) to 40 °F (4.5 °C) at the geometric center. The object can be cooled either by 35 °F (1.7 °C) cold water flowing along the surface at 60 ft/min (18.3 m/min), or by 30 °F (−1.1 °C) cold air flowing along the surface at 600 ft/min (183 m/min). Calculate the time required to cool the product to a center temperature of 40 °F (4.5 °C) and determine its mass average temperature when the center reaches 40 °F (4.5 °C). Since the ratio between the length or the width of the object to its thickness is 6:1, the heat flow will be practically one dimensional and the object can be considered to be infinitely long in the two other directions.

The film coefficient h for forced convection, over a flat plate can be calculated using equation 7.41*a* from Rohsenow and Choi (1961). Under the stated conditions the film coefficient is 2.31 and 110 Btu/hr ft^2 °F (11.3 and 537 kcal/hr m^2 °C) for the air and water, respectively. The respective N_{Bi} are 0.77 and 37. From figures 2, 3, 4 and 5 we find that :

	$f\alpha/R^2$	j_c	j_m
air	3.9	1.10	0.99
water	0.98	1.27	0.832

Solving for f we get an f value of 5.42 and 1.36 hr for the air and water, respectively. Solving equation 3 for t using the appropriate j_c we find that the time required for the geometric center to reach 40°F (4.5°C) is 4.82 and 1.65 hr for the air and water, respectively. Using the appropriate j_m, the mass average temperature, T_m, is found to be 39°F (3.9°C) and 38.2°F (3.5°C) for the air and water, respectively.

If we double the fluid velocity the corresponding film coefficient will be increased approximately by $2^{0.5}$ times. Repeating the calculations for this new case will show that the corresponding f values will be 4.32 and 1.36 hr, and the time required for the center to reach 40°F (4.5°C) will be 3.86 and 1.65 hr for the air and the water, respectively.

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NOTATION

C_p	specific heat (at constant pressure)	Btu/lb$_m$ °F
e	Napierian base (= 2.71826..)	
f	temperature response parameter; the time required for a 90% change in the temperature difference on the linear portion of the curve	hr
h	surface heat transfer coefficient	Btu/hr ft^2 °F
$J_n(\beta_n)$	Nth order Bessel function of first kind for the argument β_n	
j	lag factor $(T_a - T_1)/(T_0 - T_1)$; j_c, lag factor at the geometric center; j_m, lag factor for the mass average; j_s, lag factor at the surface	dimensionless
k	thermal conductivity	Btu/hr ft °F

L	characteristic length of product in the direction of fluid flow	ft
m	mass of product	lb_m
N_{Bi}	Biot number, hR/k	dimensionless
N_{Fo}	Fourier number $\alpha t/R^2$	dimensionless
R	radius of sphere or infinite cylinder, half the thickness of infinite slab	ft
r	variable position, distance from center of product to point of measurement	ft
T	temperature; T_0, initial temperature; T, product temperature; T_1, medium temperature; T_a, the apparent initial temperature as defined by the linear portion of the heating curve, that is, the ordinate value of the asymptote of heating curve; T_c, temperature at the geometric center; T_m, mass average temperature; T_s, surface temperature	°F
t	time	hr
U	over-all heat-transfer coefficient	Btu/hr ft^2 °F
α	thermal diffusivity	ft^2/hr
β_n	Nth root of the boundary equation for the particular shape	
π	3.14159. .	
ρ	density	lb_m/ft^3

APPENDIX

Transient heat conduction in infinite slab

$$\frac{(T-T_1)}{(T_0-T_1)} = \sum_{i=1}^{\infty} \frac{2\sin\beta_i}{\beta_i + \sin\beta_i\cos\beta_i}\cos\left(\beta_i\frac{r}{R}\right)e^{-\beta_i^2\alpha t/R^2} \qquad (A.1)$$

root equation :

$$N_{Bi} = \beta_i \tan\beta_i \qquad (A.2)$$

1st term approximation :

$$\log\frac{(T-T_1)}{(T_0-T_1)} = \frac{-\beta_1^2\alpha t}{(\ln 10)R^2} + \log\left[\left(\frac{2\sin\beta_1}{\beta_1+\sin\beta_1\cos\beta_1}\right)\cos\left(\beta_1\frac{r}{R}\right)\right] \qquad (A.3)$$

$$j = \frac{2\sin\beta_1}{\beta_1+\sin\beta_1\cos\beta_1}\cos\left(\beta_1\frac{r}{R}\right); \qquad \frac{f\alpha}{R^2} = \frac{\ln 10}{\beta_1^2}$$

$$j_c = \frac{2 \sin \beta_1}{\beta_1 + \sin \beta_1 \cos \beta_1} \tag{A.4}$$

$$j_m = \frac{2 \sin^2 \beta_1}{\beta_1(\beta_1 + \sin \beta_1 \cos \beta_1)} = j_c\left(\frac{\sin \beta_1}{\beta_1}\right) \tag{A.5}$$

$$j_s = \frac{2 \sin^2 \beta_1}{\beta_1 + \sin \beta_1 \cos \beta_1} \cos \beta_1 = j_c \cos \beta_1 \tag{A.6}$$

Transient heat conduction in infinite cylinder

$$\frac{(T - T_1)}{(T_0 - T_1)} = \sum_{i=1}^{\infty} \left(\frac{2}{\beta_i}\right) \frac{J_1(\beta_i)}{J_0^2(\beta_i) + J_1^2(\beta_i)} J_0\left(\beta_i \frac{r}{R}\right) e^{-\beta_i^2 \alpha t/R^2} \tag{A.7}$$

root equation :

$$N_{Bi} = \beta_i \frac{J_1(\beta_i)}{J_0(\beta_i)} \tag{A.8}$$

1st term approximation :

$$\log \frac{(T - T_1)}{(T_0 - T_1)} = \frac{-\beta_1^2 \alpha t}{(\ln 10) R^2} + \log \left\{\frac{2 J_1(\beta_1)}{\beta_1 [J_0^2(\beta_1) + J_1^2(\beta_1)]} J_0\left(\beta_1 \frac{r}{R}\right)\right\} \tag{A.9}$$

$$j = \frac{2 J_1(\beta_1)}{\beta_1 [J_0^2(\beta_1) + J_1^2(\beta_1)]} J_0\left(\beta_1 \frac{r}{R}\right); \quad \frac{f\alpha}{R^2} = \frac{\ln 10}{\beta_1^2}$$

$$j_c = \frac{2 J_1(\beta_1)}{\beta_1 [J_0^2(\beta_1) + J_1^2(\beta_1)]} \tag{A.10}$$

$$j_m = \frac{4 J_1^2(\beta_1)}{\beta_1^2 [J_0^2(\beta_1) + J_1^2(\beta_1)]} = j_c \frac{2 J_1(\beta_1)}{\beta_1} \tag{A.11}$$

$$j_s = \frac{2 J_1(\beta_1)}{\beta_1 [J_0^2(\beta_1) + J_1^2(\beta_1)]} J_0(\beta_1) = j_c J_0(\beta_1) \tag{A.12}$$

Transient heat conduction in sphere

$$\frac{(T - T_1)}{(T_0 - T_1)} = \sum_{i=1}^{\infty} \frac{2(\sin \beta_i - \beta_i \cos \beta_i)}{\beta_i - \sin \beta_i \cos \beta_i} \frac{\sin\left(\beta_i \frac{r}{R}\right)}{\beta_i \frac{r}{R}} e^{-\beta_i^2 \alpha t/R^2} \tag{A.13}$$

root equation :

$$N_{Bi} = 1 - \beta_i \cot \beta_i \tag{A. 14}$$

1st term approximation : (A. 15)

$$\log \frac{(T - T_1)}{(T_0 - T_1)} = \frac{-\beta_1^2 \alpha t}{(\ln 10) R^2} + \log \left\{ \left[\frac{2(\sin\beta_1 - \beta_1 \cos\beta_1)}{\beta_1 - \sin\beta_1 \cos\beta_1} \right] \left[\frac{\sin\left(\beta_1 \frac{r}{R}\right)}{\beta_1 \frac{r}{R}} \right] \right\}$$

$$j = \left[\frac{2(\sin\beta_1 - \beta_1 \cos\beta_1)}{\beta_1 - \sin\beta_1 \cos\beta_1} \right] \left[\frac{\sin\left(\beta_1 \frac{r}{R}\right)}{\beta_1 \frac{r}{R}} \right] ; \qquad \frac{f\alpha}{R^2} = \frac{\ln 10}{\beta_1^2}$$

$$j_c = \frac{2(\sin\beta_1 - \beta_1 \cos\beta_1)}{\beta_1 - \sin\beta_1 \cos\beta_1} \tag{A. 16}$$

$$j_m = j_c \frac{3}{\beta_1^3} (\sin\beta_1 - \beta_1 \cos\beta_1) \tag{A. 17}$$

$$j_s = \left[\frac{2(\sin\beta_1 - \beta_1 \cos\beta_1)}{\beta_1 - \sin\beta_1 \cos\beta_1} \right] \left[\frac{\sin\beta_1}{\beta_1} \right] = j_c \frac{\sin\beta_1}{\beta_1} \tag{A. 18}$$