EVALUATION OF SURFACE FILM HEAT TRANSFER COEFFICIENTS USING TRANSIENT METHOD

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Evaluation des coefficients de convection à l'aide d'une méthode de régime variable

RÉSUMÉ: *La mesure du coefficient de convection est très difficile en raison de l'influence de la nature et de la vitesse de l'agent de transmission de chaleur et de la forme du corps.*

On présente une méthode dans laquelle le coefficient de convection est calculé à partir des propriétés physiques et des données température-temps de traducteurs métalliques de conductivité thermique élevée. Le traducteur est habituellement de forme simple, facile à étudier analytiquement, se rapprochant de la forme du corps pour lequel le coefficient de convection doit être déterminé. Cette méthode a été appliquée à diverses installations frigorifiques pour déterminer le coefficient de convection. La grandeur de ce coefficient variait de 5.0 (convection naturelle à l'air) à 7300 kcal/h.m². °C (convection forcée dans les installations de refroidissement par eau glacée). Les résultats expérimentau concordaient avec les valeurs calculées à partir des corrélations trouvées dans la littérature pour des systèmes semblables.

INTRODUCTION

The surface heat-transfer coefficient, h, is one of the most important parameters in heat transfer problems and should be known with reasonable accuracy before design or evaluation of a heat flow system can be made. Since the film coefficient is a system property, its value is primarily a function of the physical properties and velocity of the heat transfer medium and the geometry and surface characteristics of the body. The relationship of the film coefficient and its independent variables is complex; therefore, the film coefficient is difficult to measure. The film coefficient and its independent variables are usually related through the use of dimensionless groups. Dimensionless groups, which reduce the number of variables, may arise from either a mathematical set up of the physical situation, reasoning, experience, or intuition. The equations for correlating heat transfer data are of an empirical or semi-empirical nature where the constant and exponents of the dimensionless groups are estimated, approximated or evaluated individually for each case.

There are numerous empirical or semi-empirical equations correlating the surface heat-transfer coefficient with its independent variables. Common forms are: $N_{Nu} = \psi(N_{Re}, N_{Pr})$ for forced convection, and $N_{Nu} = \psi(N_{Gr}, N_{Pr})$

for natural convection. The differences among the several correlations are, generally, in the format of the equations and the values of the various constants and exponents; these values will vary from one physical situation to another, as well as from one researcher to another. Since these constants and exponents are usually obtained by experiment, their values will depend on the geometry and size of the body, its position with respect to flow and gravity, and the flow pattern as well as the type, reliability, and sophistication of the overall experimental system.

The complex relationship of the heat transfer coefficient and its independent variables is such that even though much effort in heat transfer research has been devoted to this problem, correlations are usually available only for simple cases and simple geometries. For any one case the values for the surface heat transfer coefficient using various correlations may vary 100%.

The purpose of this paper is not to add another correlation but to present a method where the surface heat-transfer coefficient for a given system can be evaluated with adequate accuracy using high thermal conductivity metal transducers.

Basis of Method

Our approach is to construct high thermal conductivity metal transducers that will approximate both shape and size of the body whose surface heat transfer film coefficient is to be determined. For example, if the surface heat transfer film coefficient for cooling or heating an apple or potato is needed, a spherical transducer would be constructed. The transducer is exposed to the same conditions as the body. The film heat transfer coefficient can be calculated from the temperature-time history of the transducer. Since the transducer and the body are similar in shape and size, the flow pattern and momentum boundary layer around the transducer will approximate those around the body.

Theoretically the transducer can be constructed from any material, providing the thermal properties (k, ρC_p) are known and for practical purposes are independent over the experimental range of temperature. It will be shown later that, from the view point of error, high thermal conductivity materials are preferred: the higher the thermal conductivity, the smaller the likelihood of error.

To compute a film coefficient from the time-temperature history of an object, a mathematical solution must be available. Mathematical solutions are usually available when one dimensional heat flow exists, for example, spheres, infinite cylinders, or infinite slabs. Under certain conditions, mathematical solutions are available for objects which can be described as the product of two simple configurations. For example, a finite cylinder and a rectangular parallelpiped. Simple boundary conditions of the experiment are desirable; the most common conditions will be those of a body initially at a uniform temperature, T_o, suddenly exposed to a new constant temperature, T_1:

$$T = T_0 \qquad \text{at} \quad t = 0 \quad \text{for} \quad 0 \leq r \leq R$$

$$\frac{\partial T}{\partial r} = 0 \qquad \text{at} \quad r = 0 \quad \text{for} \quad t \geq 0$$

$$-k \frac{\partial T}{\partial r} = h(T - T_1) \quad \text{at} \quad r = R \quad \text{for} \quad t > 0$$

The exact solution for the three major one dimensional heat flow geometries, infinite slab, infinite cylinder and sphere having the above boundary conditions has the general from:

$$\frac{T - T_1}{T_0 - T_1} = \sum_{i=1}^{\infty} je^{-\frac{B{i}^2 \alpha t}{R^2}} \tag{1}$$

where β_i are the positive roots of the respective transcendental equations.

$$\text{infinite slab} \qquad N_{Bi} = \beta_i \tan \beta_i \tag{2a}$$

$$\text{infinite cylinder} \qquad N_{Bi} = \beta_i \frac{J_1(\beta_i)}{J_0(\beta_i)} \tag{2b}$$

$$\text{sphere} \qquad N_{Bi} = 1 - \beta_i \cot \beta_i \tag{2c}$$

There are a number of graphical methods of presenting equation 1. In McAdams (1954) the unaccomplished temperature is plotted vs. N_{Fo} with the N_{Bi} as a parameter on the graph. In Pflug *et al.* [3] the dimensionless parameter $\frac{f\alpha}{R^2}$ is plotted vs. the N_{Bi} (Fig. 1). The f in the latter presentation is the temperature response parameter which, mathematically, is the reciprocal of the absolute value of the slope of the straight asymptote to the curve produced when log (T-T_1) is plotted vs. time as shown in Fig. 2. A discussion and comparison of this method is given detail by Pflug and Kopelman [4].

The experimental temperature-time data gathered using a transducer can be plotted as in Fig. 2 and the f-value determined. Using the known thermal properties, shape and dimensions of the transducer the value of $\frac{f\alpha}{R^2}$ can be computed; the respective N_{Bi} determined from Fig. 1, and h computed from N_{Bi}.

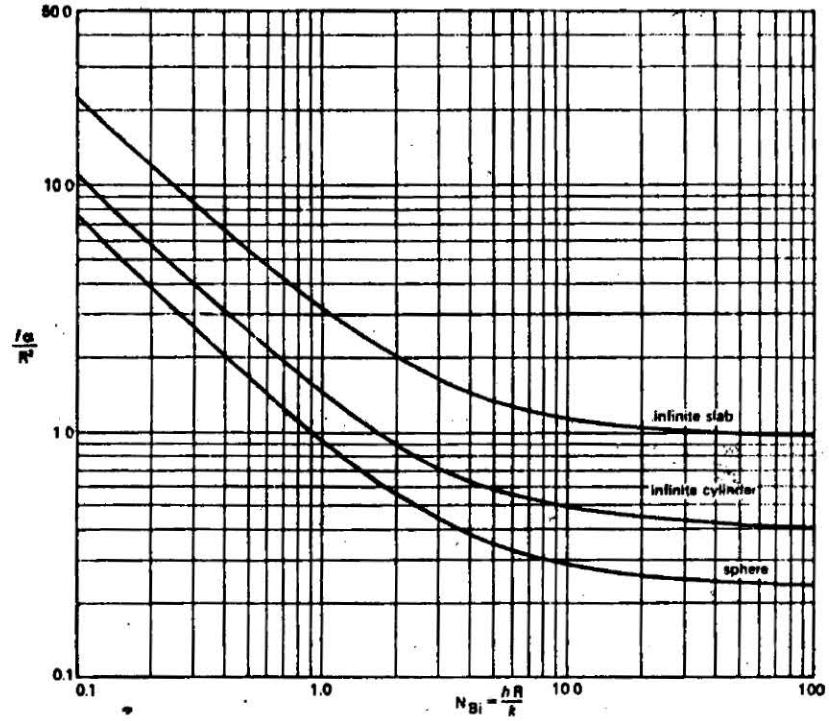

Fig. 1. — NB_i vs. $f\alpha/R^2$ for infinite slab, infinite cylinder and sphere.

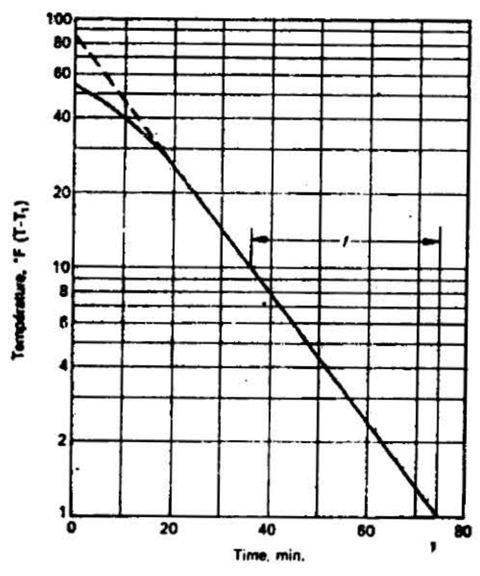

Fig. 2. — Cooling curve in convenient form for theoretical use, with temperature scale expressed in degrees difference between processing temperature T_1 and temperature of product.

PROBABLE ERROR IN THE TRANSIENT METHOD

Using the definition of the temperature response parameter, f, we can obtain from equation 1 the following relationship.

$$\frac{f\alpha}{R^2} = \frac{ln\,(10)}{\beta_1^2} \tag{3}$$

Since the N_{Bi} is the dimensionless group used to calculate the surface heat transfer coefficient, h, it is important from the viewpoint of error analysis to determine the relationship between a change in the thermal properties of the body and the N_{Bi}. Throughout this error analysis, we shall try to show that to obtain reliable results for moderate to large values of h the transducer should be fabricated from materials with a high thermal conductivity. This will be done in the following manner. Equation 3 holds for all values of N_{Bi} for any of the three basic one dimensional heat flow geometries. Taking the first derivative of α with respect to β_1 we get:

$$\frac{d\alpha}{d\beta_1} = \frac{ln(10)}{f} R^2 \left(\frac{-2}{\beta_1^3}\right) = \frac{-2\alpha}{\beta_1} \tag{4}$$

Next, by taking, for example, the derivative of the transcendental equation 2a for an infinite slab we obtain:

$$\frac{dN_{Bi}}{d\beta_1} = \tan\beta_1 + \frac{\beta_1}{\cos^2\beta_1} \tag{5}$$

This expression shows the very large rate of change of N_{Bi} with respect to β_i in the neighborhood of $\pi/2$ where both the terms $\tan\beta_1$ and $\beta^2{}_1/\cos^2\beta_1$ approach infinity. In the low N_{Bi} region:

$$\cos\beta_1 \rightarrow 1\,,$$

$$\tan\beta_1 \sim \beta_1$$

$$\text{and } \frac{d\,N_{Bi}}{d\,\beta_1} \sim 2\,\beta_1$$

The most important derivative related to the probable error in the experimentally determined value of h is probably $dN_{Bi}/d\alpha$ and its related forms which show the relationship of a change in the overall heat transfer properties of the system and a change in the thermal diffusivity. This relationship can be obtained by dividing equation 5 by equation 4:

$$\frac{dN_{Bi}}{d\alpha} = \frac{\tan\beta_1 + \dfrac{\beta_1}{\cos^2\beta_1}}{-\dfrac{2\alpha}{\beta_1}}$$

$$= -\left[\frac{\beta_1\tan\beta_1 + \left(\dfrac{\beta_1}{\cos\beta_1}\right)^2}{2\,\alpha}\right]$$

$$= -\left[\frac{N_{Bi} + \left(\dfrac{\beta_1}{\cos\beta_1}\right)^2}{2\,\alpha}\right] \tag{6}$$

Equation 6 shows the very large change of N_{Bi} with respect to α in the high N_{Bi} region and demostrates clearly how in this region, a small error in the physical properties of the transducer can cause a large error in N_{Bi}. The relationship between the relative error in the N_{Bi}, dN_{Bi}/N_{Bi} as a function of the N_{Bi} itself and the relative error of the thermal diffusivity $d\alpha/\alpha$ can be obtained by rearranging equation 6:

$$\frac{dN_{Bi}}{N_{Bi}} = -\left[\frac{N_{Bi} + \left(\dfrac{\beta_1}{\cos\beta_1}\right)^2}{2\,N_{Bi}}\right]\frac{d\alpha}{\alpha} = -S\frac{d\alpha}{\alpha} \tag{7}$$

$$S = \frac{N_{Bi} + \left(\dfrac{\beta_1}{\cos\beta_1}\right)^2}{2\,N_{Bi}} = \frac{1 + \dfrac{N_{Bi}}{Sin^2\,\beta_1}}{2} \tag{7a}$$

For the purpose of numerical approximation, equation 7 can be written in the form of finite differences:

$$\frac{\Delta N_{Bi}}{N_{Bi}} = -S\frac{\Delta\alpha}{\alpha} \tag{8}$$

Equation 8, due to its finite difference format will give only an approximation of the exact error as expressed in equation 7; as the term $\Delta N_{Bi}/N_{Bi}$ decreases it becomes more accurate since it approaches dN_{Bi}/N_{Bi}. In the high N_{Bi}

region β_1 approaches $\pi/2$ and $\sin\beta_1$ approaches 1, therefore S approaches $N_{Bi}/2$. This means that in high N_{Bi} region the probable error will be proportionately linear to N_{Bi}.

In the low N_{Bi} region S approaches 1 indicating that the relative error in N_{Bi} is equal in value (opposite in sign) to the relative error in α. The error in the determined h will depend on the source of the error in α; if due to an error in k, no error in h will result. For large values of S (high N_{Bi} system) the error $\Delta N_{Bi}/N_{Bi}$ can be evaluated only for small $\Delta\alpha/\alpha$. Evaluation of $\Delta N_{Bi}/N_{Bi}$ where S values are high for any value of $\Delta\alpha/\alpha$ (other than small values) will not only yield results with a substantial error in the determined $\Delta N_{Bi}/N_{Bi}$ but under certain conditions will result in a value of $\Delta N_{Bi}/N_{Bi}$ smaller than -1 which is physically impossible. It can be observed from equation 8 that in large values of S the $\Delta\alpha/\alpha$ must be kept small so the approximation of dN_{Bi} and $d\alpha$ by ΔN_{Bi} and $\Delta\alpha$, respectively, in the transformation from the derivative form (equation 7) to the finite difference form (equation 8) will hold.

In Fig. 3, S is plotted vs. N_{Bi} for an infinite slab (asymptotes are: $S = 1$ and $S = N_{Bi}/2$); an example of the use of the graph in Fig. 3 for evaluating the

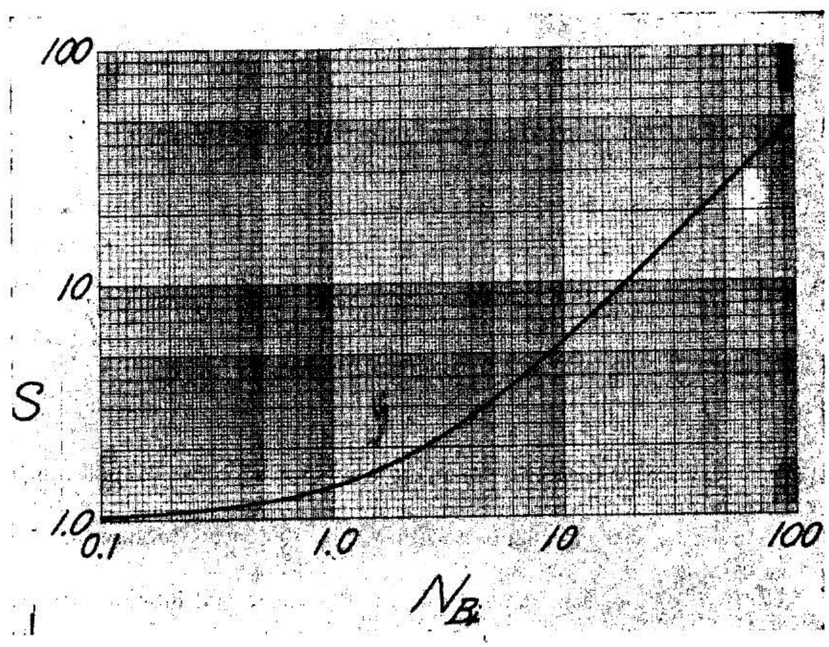

Fig. 3. — S vs. N_{Bi} for infinite slab.

error is as follows: $N_{Bi} = 1.0$, the error in α is estimated to be $+ 5\%$; from Fig. 3 $S = 1.37$ and $\Delta N_{Bi}/N_{Bi} = - 1.37 \times 0.05 = - 6.85\%$. The error in the N_{Bi} is estimated to be about $- 7\%$.

The analysis of rate of change of N_{Bi} and its relative error with respect to f and R can be repeated in the same manner outlined for α. Regarding f, it can be observed from equation 3 that both the format and the conclusions will be the same as those for α. The results of the analyses with respect to f are:

$$\frac{dN_{Bi}}{df} = - \left[\frac{N_{Bi} + \left(\frac{\beta_1}{\cos \beta_1} \right)^2}{2f} \right] \tag{9}$$

$$\frac{dN_{Bi}}{N_{Bi}} = - \left[\frac{N_{Bi} + \left(\frac{\beta_1}{\cos \beta_1} \right)^2}{2\,N_{Bi}} \right] \frac{df}{f} \tag{10}$$

Similar analyses with respect to R from equation 3 are:

$$\frac{dN_{Bi}}{dR} = \left(\tan \beta_1 + \frac{\beta_1}{\cos^2 \beta_1} \right) \sqrt{\frac{ln\,(10)}{f\alpha}} \tag{11}$$

$$\frac{dN_{Bi}}{N_{Bi}} = \frac{\left(\tan \beta_1 + \frac{\beta_1}{\cos^2 \beta_1} \right) \sqrt{\frac{ln\,(10)}{f\alpha}}}{\frac{h}{k}} \frac{dR}{R} \tag{12}$$

Again we can see the large effect of an error in f or in R on the N_{Bi} in the high N_{Bi} region.

The negative sign of equation 6, 7, 8, 9, and 10 appears because the N_{Bi} increases when α or f decreases.

Similar equations can be developed for the sphere and infinite cylinder. The same magnitude of error will result for the sphere and infinite cylinder as well as for other isotropic regular shapes such as the finite cylinder and the rectangular parallelpiped.

CONSTRUCTION AND USE OF TRANSDUCERS

We constructed transducers from either copper or aluminum. These metals were chosen because high purity commercial stocks are available, machinability, high thermal conductivity, and only moderate change in thermal properties with respect to our experimental temperature range.

This method is contingent on the accurate measurement of the temperature of the transducer. We have found that placing the thermocouple in the transducer is a critical operation. To install a thermocouple in the transducer, a 30 gage copper constantan thermocouple is placed in a square-cut end hypodermic needle which is in turn inserted into the hole drilled to the center of the transducer. We located the thermocouple at the geometric center of the transducer, but, since the temperature response parameter, *f*, is location independent, it was not necessary. When the hypodermic needle and thermocouple are in place in the hole, the thermocouple is held while the hypodermic needle is pulled back and a small amount of fine filings (same metal as the transducer) are poured into the hole. The hypodermic needle is now used to pack the filings around the thermocouple wire. This process is

Fig. 4. — 2.75 inch diameter spherical metal transducer.

repeated until the hole is filled and the thermocouple is packed tighly against the base metal of the transducer. The electrical resistance between the thermocouple wires and the transducer is used to indicate the relative contact of the thermocouple and the transducer.

Various shapes of transducers have been constructed and used in experiments where the value of the surface heat transfer coefficient was needed. Typical cases were: the evaluation of h for cans of food in an agitated water bath, jars of food heated in steam - air mixtures, apples cooled in air and in water, (transducer shown in Fig. 4), food objects being cooked in a domestic

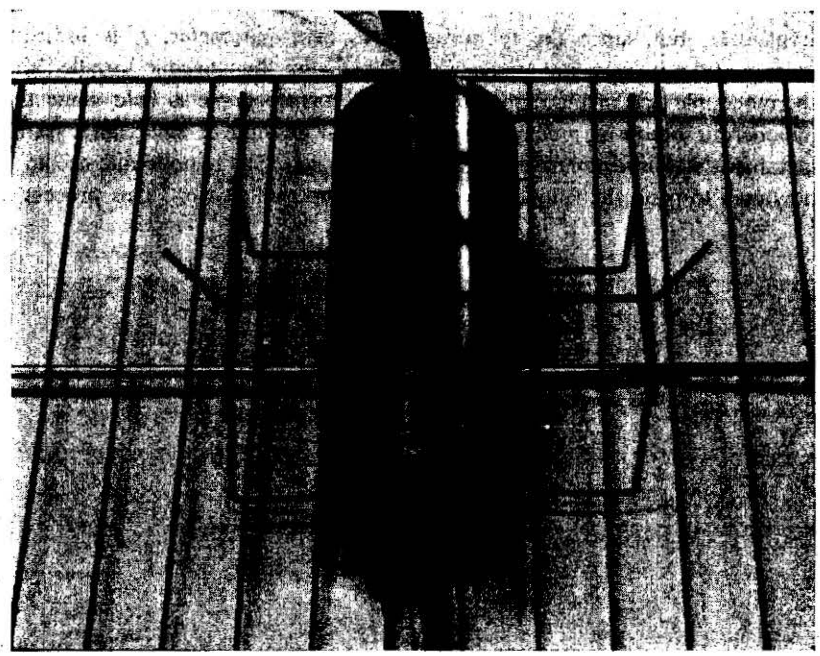

Fig. 5. — Metal transducer simulating a meat roast. This transducer was fitted with protruding thermocouples to measure air temperatures near the transducer, thermocouples to measure transducer temperatures are not visible. The transducer consists of several metal sections separated by insulation so the local heat transfer coefficient can be determined.

oven (transducers for roast and cake shown in Figs. 5 and 6). We have followed the general practice of making transducers from both aluminum and copper in order to obtain two independent measurements of h. Details of two of the applications described above are as follows:

To obtain an experimental estimate of the h for cooling apples, a 2.75 inch diameter aluminum and copper sphere (Fig. 4) were constructed and then exposed to the same experimental conditions as apples undergoing cooling tests. In experiments to determine the rate of cooling of fruit in a stack

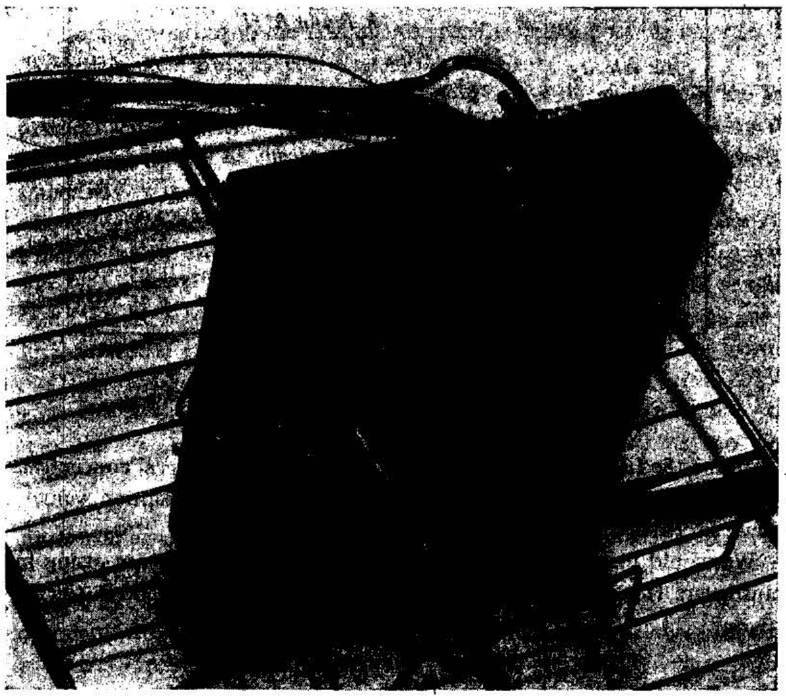

Fig. 6. — Metal transducer simulating a cake; the design is similar to the transducer in Figure 5.

tranducers were placed at different locations in the stack to measure the local h value. Our experimentally determined values were correlated with values from the literature whenever possible. In Table 1 are shown film coefficients for single spheres suspended in air determined experimentally using copper and aluminum transducers and values calculated using the correlation $N_{Nu} = 0.37\ N_{Re}^{0.6}$ (McAdams [2].

In the second case we wanted to evaluate the film heat transfer coefficient of a steam-air mixture. (Literature values for h of steam-air mixtures are very limited). Using 3 inch diameter by 4-1/2 inch high copper and aluminum transducers (that approximated the shape of the containers) we were able to

TABLE 1

Film heat transfer coefficients for a single sphere suspended in air calculated from literature correlations and determined using metal transducers

Air velocity ft/min	h, Btu/hr ft² °F		
	Experimental		Theoretical[a]
	Copper	Aluminum	
82	2.42	2.42	2.60
135	2.90	2.92	3.73
260	4.10	4.20	5.05
430	5.20	5.20	6.78

[a] $N_{Nu} = 0.37\, N_{Re}^{0.6}$ (McAdams, 1954).

determine the film coefficient of steam-air mixtures for several concentrations of air and for several heating medium velocities. The resulting values are shown in Table 2.

We would like to report that we have been able to measure, using metal transducers, film heat transfer coefficients ranging from 1 to 1500 Btu/hr ft² °F.

TABLE 2

Film heat transfer coefficient for several steam-air mixtures as a function of velocity evaluated using 4-1/2 × 3 inch cylindrical metal transducers. (Total pressure 1 atm.)

Steam-air mixture conditions		h, Btu/hr ft² °F		
Temp °F	Percent air	Velocity ft/min.		
		9	21	33
165	63.7	34	44.5	45
180	48.8	73	75	77
195	29.1	128	145	156

DISCUSSION

In the previous section it has been shown that the surface heat transfer coefficient can be evaluated unconditionally from the heating rate data and the physical and geometrical properties of the body as long as a mathematical solution is available. To avoid possible misinterpretation of the data which may lead to erroneous results it is important that we understand the overall heat transfer properties of the system and specifically the possible error in the evaluated film heat transfer coefficient.

It is advantageous when evaluating h using a transducer to be in the low N_{Bi} region since the effect of any error in the independent variables on the evaluated N_{Bi} will be of a linear nature in the low N_{Bi} region and increases exponentially in the high N_{Bi} region (Fig. 3).

Using higher thermal conductivity metals for transducer reduces the N_{Bi} and therefore the probable error. High conductivity transducers also have the advantage that in many systems (almost all air systems) the N_{Bi} will be small enough that the mathematical solution is available even if the body is not of simple configuration. (In Newtonian heating f is independent of thermal conductivity $h = ln(10)\rho Cp/f$ (volume/area), Kopelman and Pflug, 1967). We consider the thermal diffusivity to be the major probable contributor to the error of the evaluated film coefficient. The thermal diffusivity may not only lack precision but may also be temperature dependent. Errors in thermal properties due to temperature dependency over the experimental range cannot be controlled or reduced. The effect of such errors cannot be approximated, since even if the temperature relationship of the properties is known an apparent value will not be accurate since the exact solution is based on temperature independent thermal properties. Nevertheless, the error in the evaluated N_{Bi} by introducing constant values for thermal properties will be much smaller in the low N_{Bi} region compared to the high N_{Bi} region.

The value of the characteristic dimension, R, and the temperature response parameter, f, can be measured fairly accurately and usually can be improved by better experimental technique.

By observation of Fig. 1, we can develop a rule of thumb that the transducer should be constructed so that the N_{Bi} is less than 2. When the N_{Bi} is greater than 10, the curve for $f\alpha/R^2$ vs. N_{Bi} becomes less steep and N_{Bi} becomes very sentisive to any change of the $f\alpha/R^2$. This, of course, can also be observed from Fig. 3 which shows the sharp increase in the magnitude of S in the regions of the high N_{Bi}.

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NOTATION

C_p	specific heat (at constant pressure)	Btu/lb °F
e	Napierian base (= 2.71826..)	
f	temperature response parameter; the time required for a 90% change in the temperature difference on the linear portion of the curve	hr
h	surface heat transfer coefficient	Btu/hr ft^2 °F
g	gravitational constant	ft/hr^2
$J_n(\beta_n)$	Nth order Bessel function of first kind for the argument β_n	
j	lag factor $(T_a-T_1)/(T_o-T_1)$; j_c, lag factor at the geometric center; j_m, lag factor for the mass average; j_s, lag factor factor at the surface	dimensionless
k	thermal conductivity	Btu/hr ft °F
L	characteristic length of product in the direction of fluid flow	ft
N_{Bi}	Biot number, hR/k	dimensionless
N_{Fo}	Fourier number $\alpha t/R^2$	dimensionless
N_{Gr}	Grashof number, $(L^3\rho^2 g\beta\Delta T)/\mu^2$	dimensionless
N_{Pr}	Prandtl number, $C_p\mu/k$	dimensionless
N_{Re}	Reynolds number, $LV\rho/\mu$	dimensionless
R	radius of sphere or infinite cylinder, half the thickness of infinite slab	ft
r	variable position, distance from center of product to point of measurement	ft
T	temperature; T_o, initial temperature; T, product temperature; T_1, medium temperature; T_a, the apparent initial temperature as defined by the linear portion of the heating curve, that is, the ordinate value of the asymptote of the heating curve	°F
t	time	hr
V	velocity	ft/hr
α	thermal diffusivity	ft^2/hr
β	volumetric coefficient of expansion	1/°F
βn	Nth root of the boundary equation for the particular shape	
μ	voscosity of fluid	lb$_f$/ft hr
π	3.1459..	
ρ	density	lb$_m$/ft^3
ψ	indicating function	

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